

**REMARK ON CHAPTER (4)**

If  $\dim(\mathbb{V}) = n$ , and  $S = \{\underline{v}_1, \underline{v}_2, \dots, \underline{v}_k\} \subseteq \mathbb{V}$ . Then:

(1)  $k > n$ , then :-

(i)  $S$  is L.D.

(ii) $S$	may span $\mathbb{V}$	exp. $S = \{(1, 2), (2, 3), (3, 5)\}$ , since $\{(1, 2), (2, 3)\}$ span $\mathbb{V}$
	may can't span $\mathbb{V}$	exp. $S = \{(1, 2), (2, 4), (3, 6)\}$ , since there is no subset of $S$ that span $\mathbb{V}$

\*\*\*\*\*

(2)  $k < n$ , then :-

(i)  $S$  can't span  $\mathbb{V}$ .

(ii) $S$	may be L.D.	exp. $S = \{(1, 2, 3), (2, 4, 6)\}$ , since $(2, 4, 6) = 2(1, 2, 3)$
	may be L.I.	exp. $S = \{(1, 2, 3), (3, 5, 7)\}$ , since $\underline{v}_1 \neq \underline{v}_2$ .

\*\*\*\*\*

(3)  $k = n$ , then by Theorem 12 :-

$S$	(i) $S$ is L.I. $\iff S$ span $\mathbb{V}$	exp $S = \{(1, 2), (2, 3)\}$
	(ii) $S$ is L.D. $\iff S$ can't span $\mathbb{V}$	exp $S = \{(1, 2), (2, 4)\}$